First Semester Examination
2017/2018 Academic Session

January 2018

MSG368 - Sample Survey And Sampling Technique
[Tinjauan Sampel Dan Teknik Persampelan]

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **FIFTEEN (15)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA BELAS (15)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **all eight (8)** questions.

*[**Arahan :** Jawab **semua lapan (8)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

(a) Briefly describe the following sources of error in the case of a telephone only survey:

(i) Coverage error

(ii) Nonresponse error

(iii) Measurement error

(b) Identify which mode (telephone, in person , or mail) would be most desirable for household surveys, if each criterion below were of highest priority.

(i) The survey is completed quickly.

(ii) The survey costs are low for a given sample size.

(iii) The response rate is high.

(iv) Populations speaking a language different from the majority are well measured.

(v) Sampling error is minimized for a given sample size

[11 marks]

Soalan 1

(a) Terangkan secara ringkas punca-punca ralat yang berikut dalam kaji selidik yang hanya menggunakan telefon:

(i) Ralat liputan

(ii) Ralat tiada tindak balas

(iii) Ralat pengukuran

(b) Kenal pastikan mod (telefon, secara peribadi , atau mel) yang paling sesuai untuk kaji selidik isirumah, jika setiap kriteria di bawah diberi keutamaan yang paling tinggi:

(i) Kaji selidik selesai dengan cepat.

(ii) Kos kaji selidik adalah rendah untuk saiz sampel yang diberikan.

(iii) Kadar respon adalah tinggi.

(iv) Populasi yang bertutur dengan bahasa yang berbeza daripada majoriti dapat diukur dengan baik.

(v) Ralat pensampelan diminimumkan untuk saiz sampel yang diberikan.

[11 markah]

Question 2

(a) A researcher wishes to select a stratified random sample from a population with 3 strata as follows (with the usual notation):

h	N_h	s_h	c_h
1	300	30	25
2	500	40	49
3	200	20	36

(i) If the budget is RM5000 and the overheads RM800, find the allocations for each stratum.

(ii) Estimate the variance of sample mean using the allocation method from part (i).

(iii) By using proportional allocation, estimate the variance and compare it to the variance from part (ii).

(iv) Determine the total cost from the allocation of part (iii).
Explain your result.

[13 marks]

Soalan 2

(a) Seorang penyelidik ingin memilih satu sampel rawak berstrata daripala populasi dengan 3 strata seperti yang berikut (dengan notasi yang lazim):

h	N_h	s_h	c_h
1	*300*	*30*	*25*
2	*500*	*40*	*49*
3	*200*	*20*	*36*

(i) Jika belanjawan adalah RM5000 dan "overhead" adalah RM800, cari peruntukan bagi setiap kumpulan.

(ii) Anggarkan varians bagi min sampel yang menggunakan kaedah peruntukan dari bahagian (i).

(iii) Dengan menggunakan peruntukan berkadaran, anggarkan varians dan bandingkannya dengan varians dari bahagian (ii).

(iv) Tentukan jumlah kos daripada peruntukan di bahagian (iii). Jelaskan jawapan anda.

[13 marks]

Question 3

The sampling frame includes 511 farms with total acreage of 53102. A simple random sample of 9 farms was selected, and the number of acres used for growing corn and the farm size in acres were recorded for each farm. The results were as follows:

Farm	Corn (acres), y	Total acres, x
1	50	60
2	56	72
3	66	68
4	76	94
5	90	90
6	100	102
7	112	116
8	110	130
9	175	200

Given $\sum y_i = 835,\ \sum x_i = 932,\ \sum y_i^2 = 89137,\ \sum x_i^2 = 111104$ and $\sum \left(y_i - \overline{y} \right)^2 = 11667.56$

(a) Identify the variable of interest, sampling unit and any additional information associated with the units.

(b) Estimate the total number of acres used for growing corn in the region using

(i) the simple random sample estimator

(ii) the ratio estimator

(c) Calculate the estimated relative efficiency of the ratio estimator of the total compared to the regression estimator of the total.
Which estimator is better?

(d) Under what conditions is the ratio estimator useful?

(e) When is the regression estimator preferred to the ratio estimator?

[15marks]

Soalan 3

Rangka pensampelan termasuk 511 ladang dengan jumlah keluasan sebanyak 53,102 ekar. Satu sampel rawak mudah dengan 9 buah ladang telah dipilih, dan bilangan ekar yang digunakan untuk menanam jagung dan saiz ladang dalam ekar direkodkan untuk setiap ladang. Keputusannya adalah seperti berikut:

Ladang	*Jagung (ekar), y*	*Jumlah ekar, x*
1	*50*	*60*
2	*56*	*72*
3	*66*	*68*
4	*76*	*94*
5	*90*	*90*
6	*100*	*102*
7	*112*	*116*
8	*110*	*130*
9	*175*	*200*

Diberi $\sum y_i = 835,\ \sum x_i = 932,\ \sum y_i^2 = 89137,\ \sum x_i^2 = 111104$ *dan* $\sum \left(y_i - \overline{y} \right)^2 = 11667.56$

(a) *Camkan pembolehubah yang diminati, unit pensampelan dan apa-apa maklumat tambahan yang berkaitan dengan unit.*

(b) *Anggarkan jumlah ekar yang digunakan untuk menanam jagung di rantau ini dengan menggunakan*

(i) *penganggar sampel rawak mudah.*

(ii) *penganggar nisbah.*

(c) *Hitung anggaran kecekapan relatif bagi jumlah penganggar nisbah berbanding jumlah penganggar regresi. Penganggar manakah yang lebih baik?*

(d) *Di bawah syarat-syarat apakah penganggar nisbah berguna?*

(d) *Bilakah penganggar regresi lebih diutamakan daripada penganggar nisbah?*

[15 markah]

Question 4

A big company with 36 factories decided to check the condition of some equipment of which a total of 25,012 pieces are in use. A random sample of 10 factories was taken and a 10% subsample of the equipment was checked in each selected factory. The number of pieces checked and the number with signs of deterioration were as follows:

Factory	1	2	3	4	5	6	7	8	9	10
Number checked	65	82	52	91	62	69	85	73	50	76
Number defective	8	21	4	12	1	3	18	11	7	9

(a) State the type of sampling design used.

(b) Estimate the proportion of defective equipment and place a bound on the error of estimation.

(c) Estimate the total number of defective equipment.

[10 marks]

Soalan 4

Sebuah syarikat besar dengan 36 buah kilang memutuskan untuk memeriksa keadaan beberapa peralatan yang berjumlah sebanyak 25,012 buah digunakan. Satu sampel rawak daripada 10 kilang telah diambil dan 10% subsampel peralatan telah diperiksa di setiap kilang yang terpilih. Bilangan alat yang diperiksa dan bilangan dengan tanda-tanda kemerosotan adalah seperti berikut:

Kilang	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Bilangan yang diperiksa	*65*	*82*	*52*	*91*	*62*	*69*	*85*	*73*	*50*	*76*
Bilangan yang cacat	*8*	*21*	*4*	*12*	*1*	*3*	*18*	*11*	*7*	*9*

(a) Nyatakan jenis reka bentuk pensampelan yang digunakan.

(b) Anggarkan kadaran peralatan yang cacat dan tentukan batas ralat penganggaran

(c) Anggarkan jumlah peralatan yang cacat.

[10 markah]

Question 5

The University of M is interested in the average amount per week spent on groceries by married graduate students living in a high-rise apartment complex consisting of 15 floors, each floor having 10 apartments. A simple random sample of 3 floors were selected and the following data obtained.

Floor 2	Floor 6	Floor 9
RM 60	RM96	RM62
78	75	73
68	70	61
55	82	89
65	62	74
77	76	79
90	70	88
70	64	84
60	90	80
80	68	78

(a) State the type of sampling design used.

(b) Estimate the average amount (per week) spent by a married graduate student on groceries and place a bound on the error of estimation.

(c) How many floors should be included in the sample for estimating the total amount spent with a bound of RM150 on the error of estimation?

[14 marks]

Soalan 5

Universiti M berminat dengan amaun purata setiap minggu yang dibelanjakan untuk barang keperluan oleh pelajar siswazah yang telah berkahwin yang tinggal di sebuah kompleks apartmen yang terdiri daripada 15 tingkat, setiap tingkat mempunyai 10 buah apartmen. Satu sampel rawak mudah dengan 3 tingkat telah dipilih dan data diperoleh:

Tingkat 2	*Tingkat 6*	*Tingkat 9*
RM 60	*RM96*	*RM62*
78	*75*	*73*
68	*70*	*61*
55	*82*	*89*
65	*62*	*74*
77	*76*	*79*
90	*70*	*88*
70	*64*	*84*
60	*90*	*80*
80	*68*	*78*

(a) Nyatakan jenis reka bentuk pensampelan yang digunakan.

(b) Anggarkan jumlah purata (setiap minggu) yang dibelanjakan oleh pelajar siswazah berkahwin untuk barang keperluan dan tentukan batas ralat penganggaran.

(c) Berapa banyak tingkat harus dipilih sebagai sampel untuk menganggarkan jumlah amaun yang dibelanjakan dengan batas ralat penganggaran sebanyak RM150?

[15 markah]

Question 6

In a particular sector of industry, a survey is conducted in an attempt to investigate the extent of absenteeism from casual holidays, that is, holidays not connected with illness. A simple random sample of employees, from the total workforce of 36,000 were asked how many days they have taken off work, in the previous six months, as casual holidays. The results for the 1000 employees who responded were as follows:

	Number of days of casual holiday in past 6 months										
	0	1	2	3	4	5	6	7	8	9 or more	Total
Number of employees	451	162	187	112	49	21	5	11	2	0	1000

(a) Obtain point estimates, and approximate a 95% confidence interval, for

(i) the mean number of days of casual holiday taken by employees in the industry in the past six months,

(ii) the proportion of employees in the industry who have taken at least one day of casual holiday in the past six months.

(a) The organizer would, for practical reasons, much prefer to use a systematic sample for future surveys. Briefly describe systematic sampling and explain what assumptions it would require.

[13 marks]

Soalan 6

Dalam sektor industri tertentu, satu kaji selidik dijalankan dalam usaha untuk menyiasat sejauh mana ketidakhadiran daripada cuti rehat, iaitu yang tidak berkaitan dengan penyakit. Satu sampel rawak mudah pekerja-pekerja, daripada jumlah tenaga pekerja seramai 36,000 telah disoal berapa hari mereka telah mengambil cuti pada enam bulan sebelum ini, sebagai cuti rehat. Keputusan bagi 1000 pekerja yang telah menjawab adalah seperti berikut:

	Bilangan hari cuti biasa pada enam bulan lepas										
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9 atau lebih*	*Jumlah*
Bilangan pekerja	*451*	*162*	*187*	*112*	*49*	*21*	*5*	*11*	*2*	*0*	*1000*

(a) Dapatkan anggaran titik, dan anggarkan suatu selang keyakinan 95% untuk

(i) min bilangan hari bercuti rehat yang diambil oleh pekerja industri dalam tempoh enam bulan yang lalu

(ii) kadaran pekerja industri yang telah mengambil sekurang-kurangnya satu hari cuti rehat dalam tempoh enam bulan yang lalu.

(b) Penganjur mungkin, atas alasan praktikal, lebih memilih menggunakan sampel sistematik untuk kaji selidik di masa hadapan. Secara ringkas huraikan cara pensampelan sistematik, dan jelaskan andaian yang diperlukan.

[13 markah]

Question 7

An academy has N = 83 academic members. The head wants to do a salary survey selecting a circular systematic samples of n = 10 academic members. The list of members appears in **Table 1.**

(a) What should be the sampling interval *k* for this sampling design?

(b) What are the possibilities for the first random start?

(c) Suppose that a random number chosen is 53, discuss the selection procedure and obtain the estimate of the total salary for the members.

(d) Place a bound on the error of estimation of part (c).

[14 marks]

Question 7

Suatu akademi mempunyai N = 83 ahli akademik. Ketuanya ingin melaksanakan kaji selidik gaji dengan memilih sampel sistematik bulatan n = 10 ahli akademik. Senarai ahli-ahli terdapat dalam Jadual 1 (Table 1).

(a) Apakah selang pensampelan k bagi rekabentuk pensampelan ini?

(b) Apakah kemungkinan-kemungkinan bagi permulaan rawak yang pertama?

(c) Andaikan nombor rawak yang dipilih ialah 53, bincangkan prosedur pemilihan dan dapatkan anggaran jumlah gaji bagi ahli-ahli.

(d) Tentukan batas ralat penganggaran bagi bahagian (c).

[14 markah]

Question 8

(a) Show that in simple random sampling without replacement:

(i) the sample mean $\bar{y}$ is an unbiased estimator for the population mean.

(ii) $Var(\bar{y}) = \left(\frac{N-n}{N-1}\right)\frac{\sigma_y^2}{n}$

(b) Under Neyman allocation, the sample size in stratum h is

$$n_h = n\left(\frac{W_h s_h}{\sum_{h=1}^{L} W_h s_h}\right).$$

Show that

$$Var_{opt}\left(\bar{y}_{str}\right) = \frac{\sum_{h=1}^{L}\left(W_h s_h\right)^2}{n} - \frac{\sum_{h=1}^{L} W_h s_h^2}{N}$$

[10 marks]

Soalan 8

(a) Tunjukkan bahawa dalam pensampelan rawak mudah tanpa penggantian

(i) min sampel $\bar{y}$ adalah penganggar saksama bagi min populasi.

(ii) $V(\bar{y}) = \left(\frac{N-n}{N-1}\right)\frac{\sigma_y^2}{n}$

(b) Bagi peruntukan Neyman , saiz sampel dalam stratum h ialah

$$n_h = n\left(\frac{W_h s_h}{\sum_{h=1}^{L} W_h s_h}\right).$$

Tunjukkan bahawa

$$Var_{opt}\left(\bar{y}_{str}\right) = \frac{\sum_{h=1}^{L}\left(W_h s_h\right)^2}{n} - \frac{\sum_{h=1}^{L} W_h s_h^2}{N}$$

[1 0 markah]

Table 1 : Member Salaries (in RM1,000)

No.	Division	Salary
1	Eng	88
2	Medicine	45
3	Medicine	57
4	Medicine	133
5	Eng	71
6	Lit&SocSci	113
7	Medicine	65
8	Bio	47
9	Lit&SocSci	39
10	Bio	74
11	Medicine	88
12	Lit&SocSci	62
13	Lit&SocSci	49
14	Medicine	88
15	Medicine	181
16	Eng	63
17	Medicine	94
18	Eng	91
19	Medicine	60
20	Eng	55
21	Bio	55
22	Medicine	106
23	Medicine	116
24	Medicine	79
25	Lit&SocSci	61
26	Lit&SocSci	37
27	Medicine	72
28	Eng	105
29	Medicine	79
30	Medicine	61
31	Eng	86
32	Lit&SocSci	37
33	Lit&SocSci	106
34	Lit&SocSci	91
35	Bio	77
36	Medicine	90
37	Eng	71
38	Medicine	59
39	Eng	49
40	Bio	83
41	Lit&SocSci	34
42	Medicine	42

No.	Division	Salary
43	Medicine	86
44	Bio	103
45	Lit&SocSci	48
46	Eng	64
47	Eng	78
48	Medicine	53
49	Bio	85
50	Eng	61
51	Medicine	106
52	Lit&SocSci	60
53	Bio	73
54	Medicine	70
55	Medicine	32
56	Lit&SocSci	49
57	Medicine	63
58	Eng	75
59	Lit&SocSci	92
60	Medicine	107
61	Bio	57
62	Medicine	114
63	Eng	55
64	Bio	49
65	Eng	57
66	Medicine	118
67	Medicine	84
68	Eng	52
69	Medicine	64
70	Eng	75
71	Medicine	87
72	Eng	57
73	Medicine	38
74	Medicine	69
75	Eng	45
76	Medicine	83
77	Bio	78
78	Lit&SocSci	70
79	Eng	46
80	Eng	85
81	Medicine	40
82	Bio	85
83	Eng	72

Appendix /*Lampiran*

Sample	Sampel variance
$\sum_{i=1}^{n} \frac{y_i}{n}$	$\frac{s^2}{n}\left(\frac{N-n}{N}\right), s^2 = \frac{\sum_{i=1}^{n} y_i^2 - n\bar{y}^2}{n-1}$ $\frac{N-n}{2Nn(n-1)} \sum_{i}^{n-1} (y_{n+1} - y_i)^2$
$\frac{a}{n}$	$\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$
$\frac{\sum_{i=1}^{L} N_i \bar{y}_i}{N}$	$\sum_{i=1}^{L} \frac{N_i^2}{N^2}\left(\frac{N_i - n_i}{N_i}\right)\frac{s_i^2}{n_i}$ $\left(\frac{1-f}{n}\right)\sum_{i=1}^{L} W_i s_i^2 \quad ; \quad \frac{\sum_{i=1}^{L}(W_i s_i)^2}{n} - \frac{\sum_{i=1}^{L} W_i s_i^2}{N}$
$\sum_{i=1}^{n} \frac{N_i \hat{p}_i}{N}$	$\sum_{i=1}^{n} \frac{N_i^2}{N^2}\left(\frac{N_i - n_i}{N_i}\right)\frac{\hat{p}_i(1-\hat{p}_i)}{n_i - 1}$
$\frac{\bar{y}}{\bar{x}}$	$\left(\frac{N-n}{nN}\right)\left(\frac{1}{\mu_x^2}\right)\left(\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}\right)$
$\bar{y} + b(\mu_x - \bar{x})$, $b = \frac{\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$	$\left(\frac{N-n}{Nn}\right)\left(\frac{1}{n-2}\right)\left(\sum_{i=1}^{n}(y_i - \bar{y})^2 - b^2\sum_{i=1}^{n}(x_i - \bar{x})^2\right)$ $\approx \left(\frac{N-n}{Nn}\right)(S_y^2 - b^2 S_x^2)$
$\mu_x + \bar{d}$	$\left(\frac{N-n}{Nn}\right)\left(\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}\right)$
$\left(\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i}\right)$	$\left(\frac{N-n}{Nn\bar{M}^2}\right)\left(\frac{\sum_{i=1}^{n}(y_i - \bar{y}m_i)^2}{n-1}\right)$

Sampel	**Sample Variance**
$\dfrac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} m_i}$	$\left(\dfrac{N-n}{Nn\bar{M}^2}\right)\dfrac{\sum_{i=1}^{n}\left(a_i-\hat{p}m_i\right)^2}{n-1}$
$\dfrac{N}{n}\sum_{i=1}^{n} y_i$	$N^2\left(\dfrac{N-n}{Nn}\right)S_t^2$ with $S_t^2=\sum_{i=1}^{n}\dfrac{(y_i-\bar{y}_t)^2}{n-1}$
$\hat{\mu}=\dfrac{1}{n\bar{M}}\sum_{i=1}^{n} M_i\bar{y}_i$	$\left(\dfrac{N-n}{N}\right)\left(\dfrac{1}{n\bar{M}^2}\right)S_b^2+\dfrac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\dfrac{M_i-m_i}{M_i}\right)\left(\dfrac{S_i^2}{m_i}\right)$ with $S_b^2=\dfrac{\sum_{i=1}^{n}\left(M_i\bar{y}_i-\bar{M}\hat{\mu}\right)^2}{n-1}$ and $S_i^2=\dfrac{\sum_{j=1}^{m_i}\left(y_{ij}-\bar{y}_i\right)^2}{m_i-1}$
	$\left(\dfrac{N-n}{N}\right)\dfrac{MSB}{mn}+\left(1-\dfrac{m}{\bar{M}}\right)\left(\dfrac{1}{N}\right)\dfrac{MSW}{m}$
$\hat{\mu}_r=\dfrac{\sum_{i=1}^{n} M_i\bar{y}_i}{\sum_{i=1}^{n} M_i}$	$\left(\dfrac{N-n}{N}\right)\left(\dfrac{1}{n\bar{M}^2}\right)S_r^2+\dfrac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\dfrac{M_i-m_i}{M_i}\right)\left(\dfrac{S_i^2}{m_i}\right)$ with $S_r^2=\dfrac{\sum_{i=1}^{n_i}\left(M_i\bar{y}_i-\hat{\mu}_r M_i\right)^2}{n-1}$
$\hat{p}=\dfrac{\sum_{i=1}^{n} M_i\hat{p}_i}{\sum_{i=1}^{n} M_i}$	$\left(\dfrac{N-n}{N}\right)\left(\dfrac{1}{n\bar{M}^2}\right)S_r^2+\dfrac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\dfrac{M_i-m_i}{M_i}\right)\left(\dfrac{\hat{p}_i\hat{q}_i}{m_i-1}\right)$ with $S_r^2=\dfrac{\sum_{i=1}^{n_i}\left(M_i\hat{p}_i-\hat{p}M_i\right)^2}{n-1}$

Sample Size

$$n=\frac{N\sigma^2}{(N-1)D+\sigma^2} \quad ; \quad D=\frac{B^2}{4} \quad ; \quad D=\frac{B^2}{4N^2}$$

$$n=\frac{\sum_{i=1}^{L}\frac{N_i^2\sigma_i^2}{w_i}}{N^2D+\sum_{i=1}^{L}N_i\sigma_i^2} \quad ; \quad w_i=\frac{n_i}{n}$$

$$n=\frac{\left(\sum_{k=1}^{L}N_k\sigma_k/\sqrt{C_k}\right)\left(\sum_{i=1}^{L}N_i\sigma_i^2\sqrt{C_i}\right)}{N^2D+\sum_{i=1}^{L}N_i\sigma_i^2} \quad , \quad n=\frac{(C-C_o)\sum_{i=1}^{L}N_i\sigma_i/\sqrt{C_i}}{\sum_{i=1}^{L}N_i\sigma_i\sqrt{C_i}}$$

$$n_i=\frac{n\,N_i\sigma_i/\sqrt{C_i}}{\sum_{i=1}^{L}N_i\sigma_i/\sqrt{C_i}}$$

Optimal Allocation

$$n=\frac{\left(\sum_{i=1}^{L}N_i\sigma_i\right)^2}{N^2D+\sum_{i=1}^{L}N_i\sigma_i^2} \quad ; \quad n_i=n\left(\frac{N_i\sigma_i}{\sum_{i=1}^{L}N_i\sigma_i}\right)$$

Neyman Allocation

$$n=\frac{N\sum_{i=1}^{L}N_i\sigma_i^2}{N^2D+\sum_{i=1}^{L}N_i\sigma_i^2} \quad ; \quad n_i=n\left(\frac{N_i}{\sum_{i=1}^{L}N_i}\right)$$

Proportional Allocation

$$n=\frac{\sum_{i=1}^{L}N^2{}_ip_iq_i/w_i}{N^2D+\sum_{i=1}^{L}N_ip_iq_i} \quad ; \quad n_i=n\left(\frac{N_i\sqrt{p_iq_i/c_i}}{\sum_{i=1}^{L}N_i\sqrt{p_iq_i/c_i}}\right)$$

$$n=\frac{N\sigma^2}{ND+\sigma^2} \quad ; \quad D=\frac{B^2\mu_x^2}{4} \quad ; \quad D=\frac{B^2}{4} \quad ; \quad D=\frac{B^2}{4N^2}$$

$$n=\frac{N\sigma_r{}^2}{ND+\sigma_r{}^2} \quad ; \quad n=\frac{N\sigma_t{}^2}{ND+\sigma_t{}^2}; n=\frac{N\sigma_p{}^2}{ND+\sigma_p{}^2}$$

$$D=\frac{B^2\left(\bar{M}\right)^2}{4} \quad ; \quad D=\frac{B^2}{4N^2}$$

Intra class correlation coefficient
$\rho_w = \frac{2}{\sigma^2 nk(n-1)} \sum_{i=1}^{k} \sum_{j<u}^{n} (y_{ij} - \mu)(y_{iu} - \mu)$

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